g/cm^3. Intensity curves are shown for isothermal shock layers with thicknesses from 0.1 to 100 cm. The blackbody intensity is also shown. Individual data points on the graphs are from Nelson and Goulard[1] and Lasher et al.[2] The results of this study agree with the previous results of Nelson and Goulard to about 15%.

The step-wise gray approximation of the radiative absorption coefficient significantly reduces the numerical computer time required to make nongray radiation calculations while retaining a high degree of accuracy. The step-wise gray model is not limited to atomic hydrogen plasmas. It can be extended to include plasmas made up of several species by deriving step models for each of the components of the plasma and by keeping the frequency intervals the same. It can be applied easily to nonisothermal plasmas by representing the temperature profile by a series of isothermal steps. In addition it also naturally includes line wing overlapping.

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Gravitational Interaction Torques

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IN the analysis of motions of artificial satellites, particularly gravity-stabilized satellites with auxiliary damping bodies, torques resulting from gravitational attraction between parts of a satellite are frequently neglected. This Note is concerned with whether or not gravitational interaction torques are necessarily much smaller than gravity torques exerted on parts of a satellite by the Earth's gravitational field.

Consider the situation depicted in Fig. 1. When the largest dimension of the satellite S is small compared to the orbit radius R, the earth gravity torque T_E acting on S is given approximately by Ref. 1.

$$T_E = (3G\varepsilon/2R^3)(I_2 - I_1)\sin 2\phi \tag{1}$$

where G is the universal gravitational constant, ε is the mass of the Earth, and I_1 and I_2 are the moments of inertia of S about the centroidal principal axes "1" and "2." Typically, R is of the same order of magnitude as the radius of the Earth. Hence, ε/R^3 is of the same order of magnitude as ρ_E, the mean mass density of the Earth, and T_E is thus of the same order

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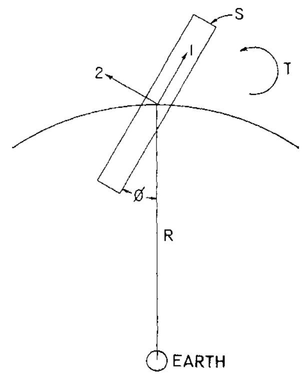

Fig. 1 Earth satellite.

of magnitude as $G\rho_E(I_2 - I_1)$. Suppose now that S is one part of a two-body satellite, and that ρ_B is the mean mass density of the second part—say, a damper boom. Then, if one uses the expression $G\rho_B(I_2 - I_1)$ to estimate the order of magnitude of the gravitational torque T_B exerted on S by the boom, one is forced to conclude that T_B is of the same order of magnitude as T_E, since ρ_B may be expected to be of the same order of magnitude as ρ_E. However, this argument has a fundamental flaw: it ignores the fact that the distances between particles of the two bodies comprising the satellite cannot be large compared with the dimensions of the satellite, which means that an assumption made in the derivation of Eq. (1) cannot be fulfilled, and the expression $G\rho_B(I_2 - I_1)$ cannot be used with impunity.

As will be seen presently, the answer to the question at hand is highly system-dependent. Consider, for example, the simple satellite configuration suggested by Fig. 2, where A designates a body consisting of a rigid, massless rod of length $2a$ carrying a particle of mass $\alpha/2$ at each end, and B is a rigid, massless rod of length $2b$ with a particle of mass $\beta/2$ at each end. The rods are joined at their midpoints by a hinge, and θ is the radian measure of the angle between rod B and a line perpendicular to rod A.

Using the inverse square law of gravitational attraction, one can express the interaction torque T_I, that is, the moment about point 0 of the forces exerted by the particles of A on those of B, or vice-versa, as

$$T_I = T^*\left[\left(\frac{a}{b} + \frac{b}{a} - 2\sin\theta\right)^{-3/2} - \left(\frac{a}{b} + \frac{b}{a} + 2\sin\theta\right)^{-3/2}\right]\cos\theta \tag{2}$$

where T^* is defined as

$$T^* = G\alpha\beta/2(ab)^{1/2} \tag{3}$$

In Fig. 3, T_I/T^* is plotted against θ for four values of a/b. The peak value of T_I is seen to be dependent on a/b, values of this ratio close to unity giving rise to large peak torques.

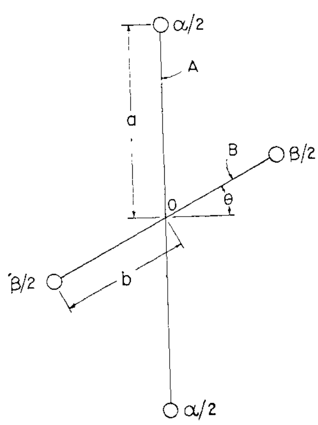

Fig. 2 Simple satellite configuration.

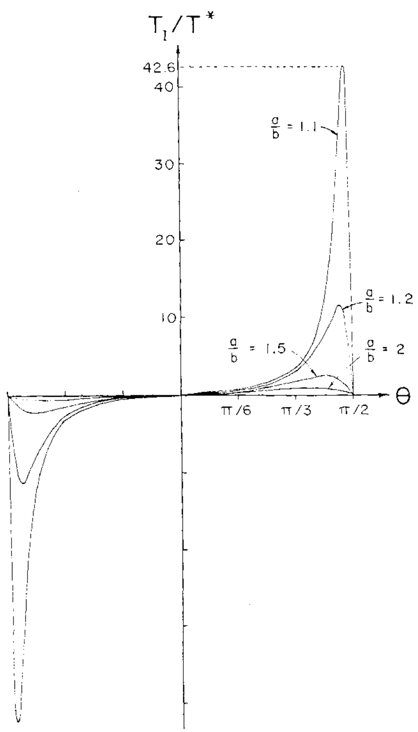

Fig. 3 Gravitational interaction torque.

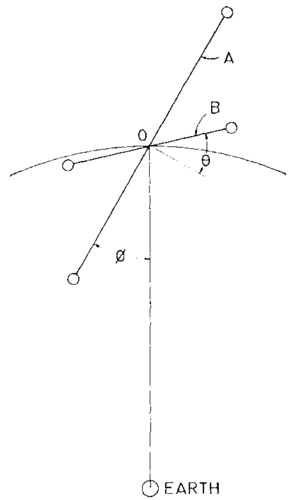

Fig. 4 Simple satellite in orbit.

To assess the relative importance of T_I, we place the system in an Earth orbit as indicated in Fig. 4, and let T_E denote the gravity torque exerted by the Earth on body A, so that, in accordance with Eq. (1), $T_E = (3G\varepsilon/2R^3)\,\alpha a^2 \sin 2\phi$. Next, we define a quantity r as

$$r = R(\beta/\varepsilon)^{1/3} \tag{4}$$

and express T_I/T_E as $T_I/T_E = (T_I/T^*)(T^*/T_E)$ or, using Eq. (3), as

$$T_I/T_E = (r/b)^3 (T_I/T^*)/3(a/b)^{5/2} \sin 2\phi \tag{5}$$

It is now easy to see that T_I can exceed T_E, even when T_E is as large as possible. Take $\phi = \pi/4$, $r/b = 1/2$, $a/b = 1.1$, and $T_I/T^* = 42.6$ (the last is the peak value of T_I/T^* when $a/b = 1.1$), as shown in Fig. 3. Then, from Eq. (5), $T_I/T_E = 1.4$.

Because r/b was set equal to $\frac{1}{2}$ "with malice aforethought" (that is, to make $T_I > T_E$, rather than to simulate a typical satellite), the physical significance of the result just obtained remains to be elucidated.

Note that, from Eq. (4)

$$r/b = (R/b)(\beta/\varepsilon)^{1/3} \tag{6}$$

and take $R = 7 \times 10^6$ m, $\varepsilon \approx 6 \times 10^{24}$ kg, and $\beta = 10$ kg, these being reasonable values for an Earth satellite. For $r/b = \frac{1}{2}$, Eq. (6) then leads to $b \approx 0.08$ m, which means that the satellite is somewhat smaller than the typical gravity-stabilized Earth satellite. If, on the other hand, we make $b = 10$ m and solve Eq. (6) for β, we find that $\beta \approx 1.6 \times 10^6$ kg, which means that we are dealing with an extraordinarily massive satellite.

For values of b and β representative of a typical gravity-stabilized satellite, the interaction torque is found to be indeed negligible. For example, with $R = 7 \times 10^6$ m, $\varepsilon = 6 \times 10^{24}$ kg, $b = 10$ m, and $\beta = 10$ kg, Eq. (6) gives $r/b = 8.3 \times 10^{-3}$; and with $\phi = \pi/4$, $a/b = 1.1$, and $T_I/T^* = 42.6$, Eq. (5) yields $T_I/T_E \approx 6.4 \times 10^{-6}$.

The conclusion that emerges from these considerations is that for relatively light but large satellites, gravitational interaction torques tend to be negligible in comparison with the gravity torque of the Earth, whereas in the case of a relatively massive but small structure, gravitational interaction torques can play a predominant role.

Reference

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